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Cross Modulation in the Scamp Signal Processor

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Abstract: When two or more signals of different frequencies are fed simultaneously into the input of a limiter, cross-modulation products exist in the output. Such a situation occurs in the Scamp (Single-Channel Monopulse Processor) signal processor when the sum signal and both difference signals (azimuth and elevation) of a monopulse system are processed in a common limiter. The resultant cross modulation can produce deleterious effects, even though the spectra of the three input signals do not overlap. By the use of approximation methods, formulas are developed for the limiter outputs, showing the dependence of the cross-modulation terms on the input amplitudes and phases of both difference channels. Curves are presented illustrating the amount of cross modulation for several arrangements of channel spacing. The largest errors occur when the difference channels are symmetrically located on either side of the reference channel. The errors are substantially reduced by an unsymmetrical or a noncontiguous channel orientation. Experimental results on a simulated Scamp processor agree favorably with the theory.

INTRODUCTION

A new monopulse processing technique called Scamp (Single-Channel Monopulse Processor) was introduced by Rubin and Kamen (1) in which a difference signal is normalized by the sum signal simultaneously in a single channel. An extension of this technique was also suggested whereby the information was processed by feeding all three channels, *i.e.*, the sum signal and both difference signals (azimuth and elevation), on three separate carriers, into a common wide-band i-f amplifier. The signals are next hard limited and then separated by three narrow-band filters. The amplitude normalization occurs in the limiter, an example of the well-known weak-signal suppression (2). It is the purpose of this present report to develop the theory governing the behavior of Scamp in its fullest embodiment using three carrier frequencies and to demonstrate the existence and effects of deleterious cross modulation between the three signals.

ANALYSIS OF SCAMP WITH TWO DIFFERENCE SIGNALS

The fundamental implementation of the Scamp technique required to normalize both difference signals simultaneously is shown in Fig. 1. The theoretical analysis of this system which follows will be developed on a constant-amplitude, continuous-wave basis as was the original analysis by Rubin and Kamen. In fact, the theory will proceed in a manner analogous to Rubin and Kamen (1), with the complication of an additional difference signal. Let us define the inputs (Fig. 1) as

$$S_a(\omega_a, t) = A_a \cos(\omega_a t + \phi_a)$$

$$S_s(\omega_s, t) = A_s \cos(\omega_s t + \phi_s)$$

$$S_e(\omega_e, t) = A_e \cos(\omega_e t + \phi_e), \tag{1}$$

where A_a represents the amplitude of the azimuth difference signal, ω_a the azimuth angular carrier frequency, ϕ_a an arbitrary epoch angle, *etc.* Provided the i-f amplifier and summing

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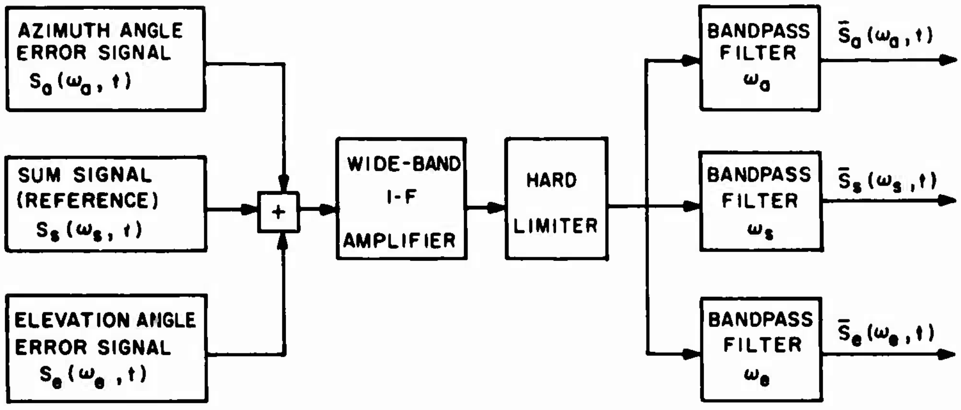

Fig. 1 — Scamp processor for simultaneous normalization of two difference signals

circuit are linear, we can consider the input to the limiter as the sum of the three expressions appearing at the input of the signal processor. Then the limiter input is

$$S(t) = A_a \cos(\omega_a t + \phi_a) + A_s \cos(\omega_s t + \phi_s) + A_e \cos(\omega_e t + \phi_e). \tag{2}$$

We can rewrite Eq. (2) as follows:

$$S(t) = A_a \cos[\omega_s t + (\omega_a - \omega_s)\, t + \phi_a] + A_s \cos(\omega_s t + \phi_s)$$
$$+ A_e \cos[\omega_s t + (\omega_e - \omega_s)\, t + \phi_e], \tag{3}$$

which can be expanded by trigonometric identities and rearranged to be of the form

$$S(t) = C \cos \omega_s t - D \sin \omega_s t, \tag{4}$$

where we define

$$C = A_a \cos[(\omega_a - \omega_s)\, t + \phi_a] + A_s \cos \phi_s + A_e \cos[(\omega_e - \omega_s)\, t + \phi_e] \tag{5}$$

and

$$D = A_a \sin[(\omega_a - \omega_s)\, t + \phi_a] + A_s \sin \phi_s + A_e \sin[(\omega_e - \omega_s)\, t + \phi_e]. \tag{6}$$

Equation (4) can also be cast in the form

$$S(t) = \sqrt{C^2 + D^2} \cos(\omega_s t + \psi), \tag{7}$$

where

$$\cos \psi = \frac{C}{\sqrt{C^2 + D^2}} \text{ and } \sin \psi = \frac{D}{\sqrt{C^2 + D^2}} \cdot \tag{8}$$

From Eq. (7) we observe that the input signal can be thought of as an amplitude and phase-modulated signal of unmodulated carrier phase $\omega_s t$.

It has been shown by Davenport and Root (3) that the output of an ideal bandpass limiter, when driven by a narrow-band signal, is a signal with a phase modulation identical to that of the

input signal and a constant amplitude ($4T/\pi$), where "T" is the threshold of the limiter. Hence, for the input Eq. (7) we may write the limiter output as

$$L(t) = \frac{4T}{\pi} \cos\,(\omega_s t + \psi). \tag{9}$$

Expanding $\cos\,(\omega_s t + \psi)$ and using Eq. (8) we have

$$L(t) = \frac{4T}{\pi} \frac{C \cos \omega_s t - D \sin \omega_s t}{\sqrt{C^2 + D^2}}. \tag{10}$$

Introducing the expressions for C and D from Eqs. (5) and (6) we have the limiter output as:

$$L(t) = \frac{4T}{\pi} \frac{\left[\frac{A_a}{A_s} \cos\,(\omega_a t + \phi_a) + \cos\,(\omega_s t + \phi_s) + \frac{A_e}{A_s} \cos\,(\omega_e t + \phi_e)\right]}{\left\{\left[\left(\frac{A_a}{A_s}\right)^2 + 1 + \left(\frac{A_e}{A_s}\right)^2\right] + 2\left[\frac{A_a}{A_s} \cos\,[(\omega_s - \omega_a)t + (\phi_s - \phi_a)] + \frac{A_a A_e}{A_s^2} \cos\,[(\omega_a - \omega_e)t + (\phi_a - \phi_e)] + \frac{A_e}{A_s} \cos\,[(\omega_s - \omega_e)t + (\phi_s - \phi_e)]\right]\right\}^{1/2}}. \tag{11}$$

For mathematical expedience we write Eq. (11) as

$$L(t) = \frac{4T}{\pi} \left[\frac{F}{(u + v)^{1/2}}\right] \tag{12}$$

where

$$F = \frac{A_a}{A_s} \cos\,(\omega_a t + \phi_a) + \cos\,(\omega_s t + \phi_s) + \frac{A_e}{A_s} \cos\,(\omega_e t + \phi_e)$$

$$u = \left(\frac{A_a}{A_s}\right)^2 + 1 + \left(\frac{A_e}{A_s}\right)^2$$

$$v = 2\left[\frac{A_a}{A_s} \cos\,[(\omega_s - \omega_a)t + (\phi_s - \phi_a)] + \frac{A_a A_e}{A_s^2} \cos\,[(\omega_a - \omega_e)t + (\phi_a - \phi_e)] + \frac{A_e}{A_s} \cos\,[(\omega_s - \omega_e)t + (\phi_s - \phi_e)]\right]. \tag{13}$$

We may expand Eq. (12) in terms of v/u provided $u > v$. This is a valid expansion for small values of A_a/A_s and A_e/A_s, in the order of 0.3 or less, regardless of the relative phases of the components of v. If in fact these phases can be considered random, then the expansion is valid up to $A_a/A_s = A_e/A_s \cong 0.5$. Expansion of Eq. (12) yields

$$L(t) = \frac{4T}{\pi} F u^{-1/2} \left(1 - \frac{v}{2u} + \frac{3v^2}{8u^2} - \frac{5v^3}{16u^3} + \frac{35v^4}{128u^4} - \frac{63v^5}{256u^5} + \ldots\right) \tag{14}$$

from which the frequency components are determined upon substitution of the relations in Eq. (13). A partial expansion of the terms in Eq. (14) is presented in the Appendix, up to and including fourth powers of A_a/A_s and A_e/A_s. It is seen from these expansions that terms containing sums and differences of almost all multiples of the three carrier input frequencies exist. Some of these give rise to cross modulation at the limiter output.

EQUALLY SPACED SYMMETRICAL CHANNELS

In order to study the cross-modulation effects in the limiter, the spacing and orientation of the three input channels must be established. Since in general it is difficult to obtain wide-band i-f amplifiers with uniform amplitude and linear phase response, one of the most expedient choices is to use equal spacing of the three signal frequencies and use contiguous channels symmetrically oriented with respect to the sum channel. Such an arrangement is shown in Fig. 2, where $\omega_s - \omega_a = \omega_e - \omega_s$. It will be shown that this particular choice is not a good one, since it leads to a large amount of cross modulation in the limiter output.

Let us examine the expansion in the Appendix to determine which terms contribute to the output at the angular frequency ω_a. Note that since $\omega_s - \omega_a = \omega_e - \omega_s$, terms such as $2\omega_s - \omega_e$, $2\omega_s - \omega_e - 2\omega_a$, *etc.*, all exhibit the angular frequency ω_a. Collecting those terms only, from the Appendix, we find in Eq. (A8) that the output at ω_a is given by

$$\begin{aligned} L_1(t)|\omega_a = \frac{4T}{\pi u^{1/2}} \Bigg\{ & \left[\frac{A_a}{A_s} - \frac{A_a}{2uA_s}\left(1 + \frac{A_e^2}{A_s^2}\right) + \frac{3A_a}{4u^2A_s}\left(\frac{A_a^2}{A_s^2} + \frac{3A_e^2}{A_s^2}\right)\right. \\ & \left. - \frac{15A_a}{16u^3A_s}\left(\frac{A_a^2}{A_s^2} + 2\frac{A_e^2}{A_s^2}\right)\right] \cos\,(\omega_a + \phi_a) \\ & - \left[\frac{A_e}{2uA_s} - \frac{3A_e}{4u^2A_s}\left(2\frac{A_a^2}{A_s^2} + \frac{A_e^2}{2A_s^2}\right) + \frac{15A_e}{16u^3A_s}\left(\frac{A_e^2}{A_s^2} + 2\frac{A_a^2}{A_s^2}\right)\right] \cos\,(\omega_a t + \phi_a + \phi_0) \\ & + \frac{3}{8}\left[\frac{A_a^2A_e}{u^2A_s^3}\left(3 - \frac{5}{2u}\right)\right] \cos\,(\omega_a t + \phi_a - \phi_0) \\ & - \frac{15}{16}\left[\frac{A_aA_e^2}{u^3A_s^3}\right] \cos\,(\omega_a t + \phi_a + 2\,\phi_0)\Bigg\}, \end{aligned} \tag{15}$$

where u has been defined in Eq. (13) and $\phi_0 = 2\,\phi_s - \phi_a - \phi_e$. An inspection of Eq. (15) shows that for $A_a = 0$, the output $L_1(t)|\omega_a$ is a function of A_e, thus indicating the presence of cross

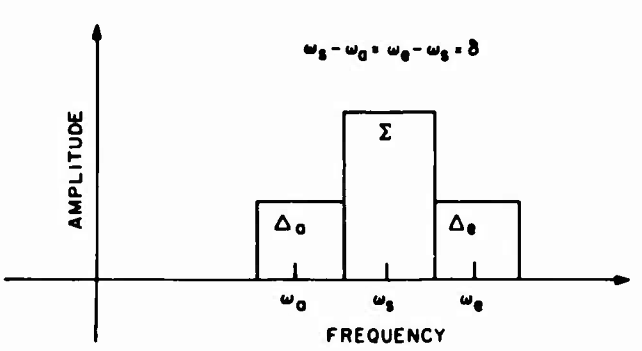

Fig. 2 — Equally spaced symmetrical channels

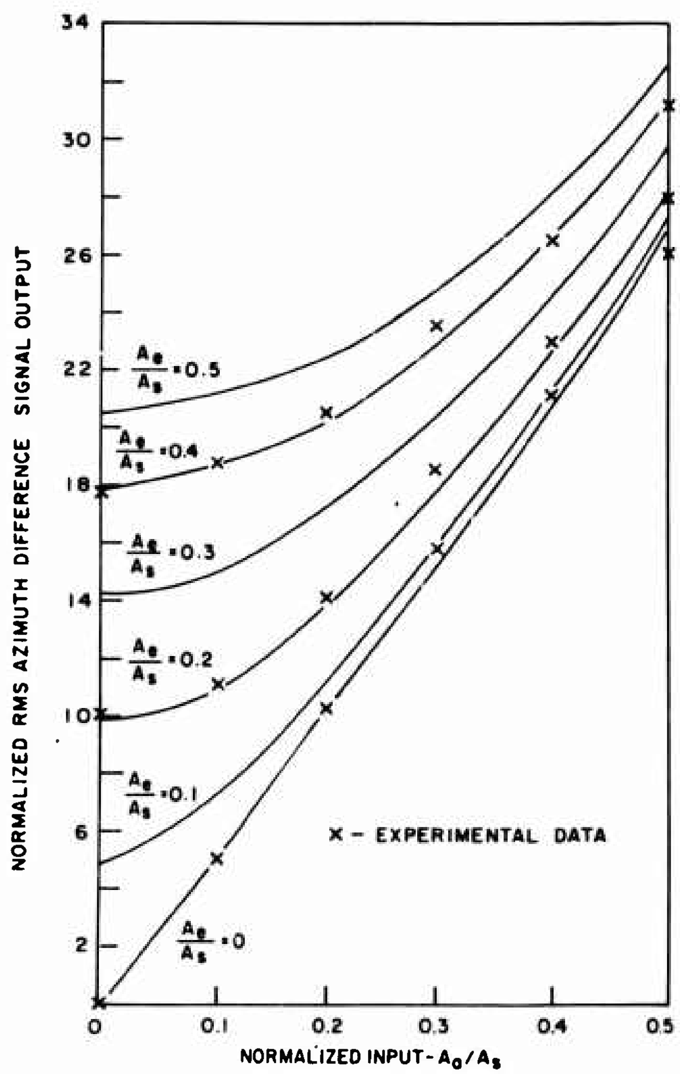

Fig. 3 — Illustration of cross-modulation effect for symmetrical equispaced channels

modulation. The four bracketed terms in Eq. (15) are noted to have the same angular frequency with phase angles which differ by multiples of ϕ_0, a function of random epoch angles. Thus a representative estimate of the output at ω_a should be the rms value of the four terms in Eq. (15). A plot of this rms output is presented in Fig. 3. The essentially linear curve for $A_e/A_s = 0$ represents the correct angle-error output, which approaches zero as $A_a/A_s \to 0$ as it should. However, it is seen that for A_e/A_s not equal to zero, the azimuth difference-signal output does not approach zero as $A_a/A_s \to 0$, but instead exhibits an error which is a function of the elevation difference signal. It is readily understood that this cross talk is a very undesirable situation.

The symmetry in the arrangement of Fig. 2, plus a very quick look at the expansion in the Appendix, permits the use of Eq. (15) for the elevation-error output by merely interchanging ω_a and ω_e. Hence, Fig. 3 also can be interpreted as the cross modulation in the elevation channel as a function of the azimuth signal.

NONSYMMETRIC CHANNEL SPACING

A number of methods for substantially reducing this cross modulation could be used. The requirement is that the arrangement or frequency spacing of the difference channels must be

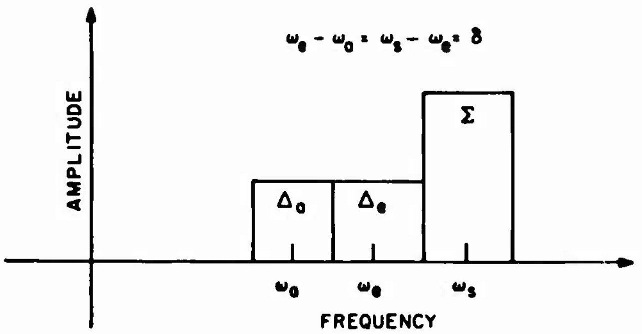

Fig. 4 — Nonsymmetric channel spacing

such as to minimize the mutual interference at the output. Most methods would require more bandwidth than the first one presented above, using equispaced symmetric channels. However, one very simple method* is the use of the arrangement shown in Fig. 4. Here again we have equal channel spacing, but both difference channels are on the same side of the sum channel. In this case, since the two difference channels are not symmetrically disposed about the reference channel, the amount and effect of cross modulation is not the same in the two difference channels.

Referring again to the Appendix, we find in Eq. (A9) the output at ω_a:

$$L_2(t)|\omega_a = \frac{4T}{\pi u^{1/2}}\left\{\left[\frac{A_a}{A_s} - \frac{A_a}{2uA_s}\left(1+\frac{A_e^2}{A_s^2}\right) + \frac{3A_a}{4u^2A_s}\left(\frac{A_a^2}{A_s^2}+\frac{3A_e^2}{A_s^2}\right)\right.\right.$$
$$\left. - \frac{15A_a}{16u^3A_s}\left(\frac{A_a^2}{A_s^2}+\frac{2A_e^2}{A_s^2}\right)\right]\cos(\omega_a t + \phi_a)$$
$$+\left[-\frac{A_e}{2uA_s} + \frac{3A_e}{8u^2A_s}\left(4\frac{A_a^2}{A_s^2}+1\right) - \frac{15A_e^2}{16u^3A_s^2}\left(5\frac{A_a^2}{A_s^2}+\frac{A_e^2}{A_s^2}\right)\right.$$
$$\left.+\frac{35A_e^2}{32u^4A_s^2}\left(3\frac{A_a^2}{A_s^2}+\frac{A_e^2}{A_s^2}\right)\right]\cos(\omega_a t + \phi_a - \phi_0)$$
$$\left.+\left[\frac{9A_a^2A_e^2}{8u^2A_s^4} - \frac{45A_a^2A_e^2}{16u^3A_s^4} + \frac{105A_a^2A_e^2}{64u^4A_s^4}\cos(\omega_a t + \phi_a + \phi_0)\right]\right\}. \tag{16}$$

In Fig. 5 is shown the rms output from Eq. (16) for the azimuth channel. It is noted that there is still a small residual cross modulation present, but it is far less than in the symmetric case and perhaps is sufficiently small as to be considered innocuous for most applications.

The formula from Eq. (A10) for the output at ω_e is:

$$L_2(t)|\omega_e = \frac{4T}{\pi u^{1/2}}\left\{\left[\frac{A_e}{A_s} - \frac{A_e}{2uA_s}\left(\frac{A_a^2}{A_s^2}+1\right) + \frac{3A_e}{4u^2A_s}\left(\frac{A_e^2}{A_s^2}+\frac{3A_a^2}{A_s^2}\right)\right.\right.$$
$$\left. - \frac{15A_e}{16u^3A_s}\left(\frac{A_e^2}{A_s^2}+\frac{2A_a^2}{A_s^2}\right)\right]\cos(\omega_e t + \phi_e)$$
$$+\left[-\frac{A_aA_e}{uA_s^2} + \frac{3A_aA_e}{4u^2A_s^2}\left(\frac{A_a^2}{A_s^2}+\frac{A_e^2}{A_s^2}+1\right) - \frac{15A_aA_e}{4u^3A_s^2}\left(\frac{A_e^2}{A_s^2}+\frac{A_a^2}{A_s^2}\right)\right.$$
$$\left.+\frac{105A_aA_e}{32u^4A_s^2}\left(\frac{A_a^2}{A_s^2}+\frac{A_e^2}{A_s^2}\right)\right]\cos(\omega_e t + \phi_e + \phi_0)$$
$$\left.+\left[\frac{3A_aA_e^3}{4u^2A_s^4} - \frac{15A_aA_e^3}{8u^3A_s^4} + \frac{35A_aA_e^3}{32u^4A_s^4}\right]\cos(\omega_e t + \phi_e - \phi_0)\right\}. \tag{17}$$

*Private communication from W. L. Rubin to NRL.

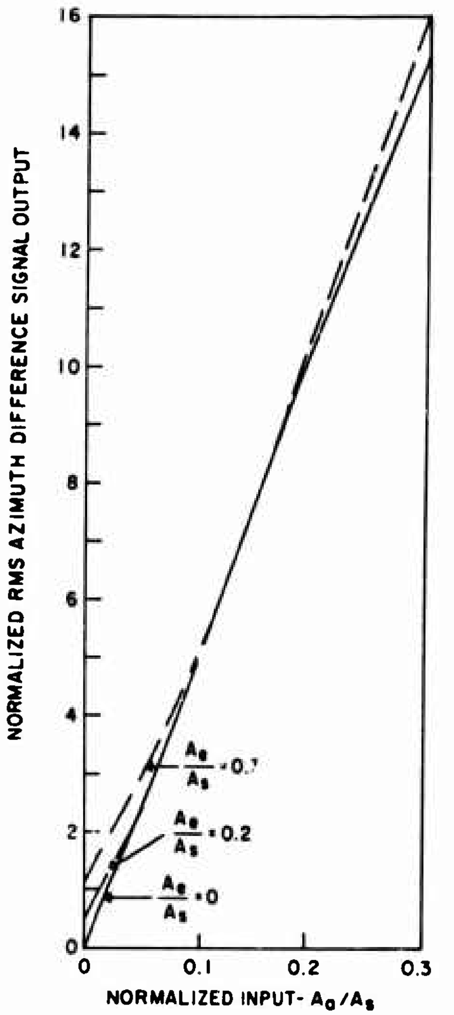

Fig. 5 – Illustration of cross-modulation effect in azimuth channel for unsymmetrically spaced channels

Calculations using Eq. (17) show that there is essentially no cross modulation present in the elevation channel.

UNEQUAL CHANNEL SPACING

A second method for reducing cross modulation is the use of unequally spaced channels. One example is shown in Fig. 6. Here the spacing of the difference channels is 2:1 with respect to the sum reference channel. From Eq. (A11) in the Appendix the output at ω_a is given by:

$$\begin{aligned} L_3(t)|\omega_a = \frac{4T}{\pi u^{1/2}} \Bigg\{ & \left[\frac{A_a}{A_s} - \frac{A_a}{2uA_s}\left(1 + \frac{A_e^2}{A_s^2}\right) + \frac{3A_a}{4u^2A_s}\left(\frac{A_a^2}{A_s^2} + \frac{3A_e^2}{A_s^2}\right)\right. \\ & \left. - \frac{15A_a}{16u^3A_s}\left(\frac{A_a^2}{A_s^2} + \frac{2A_e^2}{A_s^2}\right)\right] \cos\,(\omega_a t + \phi_a) \\ & + \left[\frac{3A_aA_e}{4u^2A_s^2} - \frac{15A_aA_e}{8u^3A_s^2}\left(\frac{A_a^2}{A_s^2} + \frac{A_e^2}{A_s^2}\right) + \frac{105A_aA_e}{32u^4A_s^2}\left(\frac{A_a^2}{A_s^2} + \frac{A_e^2}{A_s^2}\right)\right] \cos\,(\omega_a t + \phi_a + \phi_0) \\ & + \left[-\frac{5A_a^3A_e}{4u^3A_s^4} + \frac{35A_a^3A_e}{32u^4A_s^4}\right] \cos\,(\omega_a t + \phi_a - \phi_0)\Bigg\}. \end{aligned} \tag{18}$$

Figure 7 shows that there is no significant cross modulation in the azimuth channel for this 2:1 spacing.

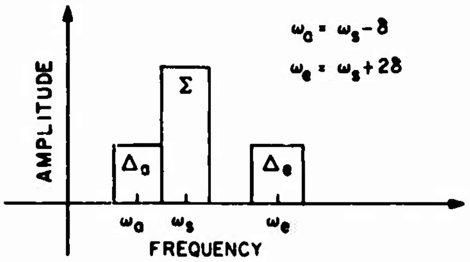

Fig. 6 – Symmetrical but unequally spaced channels

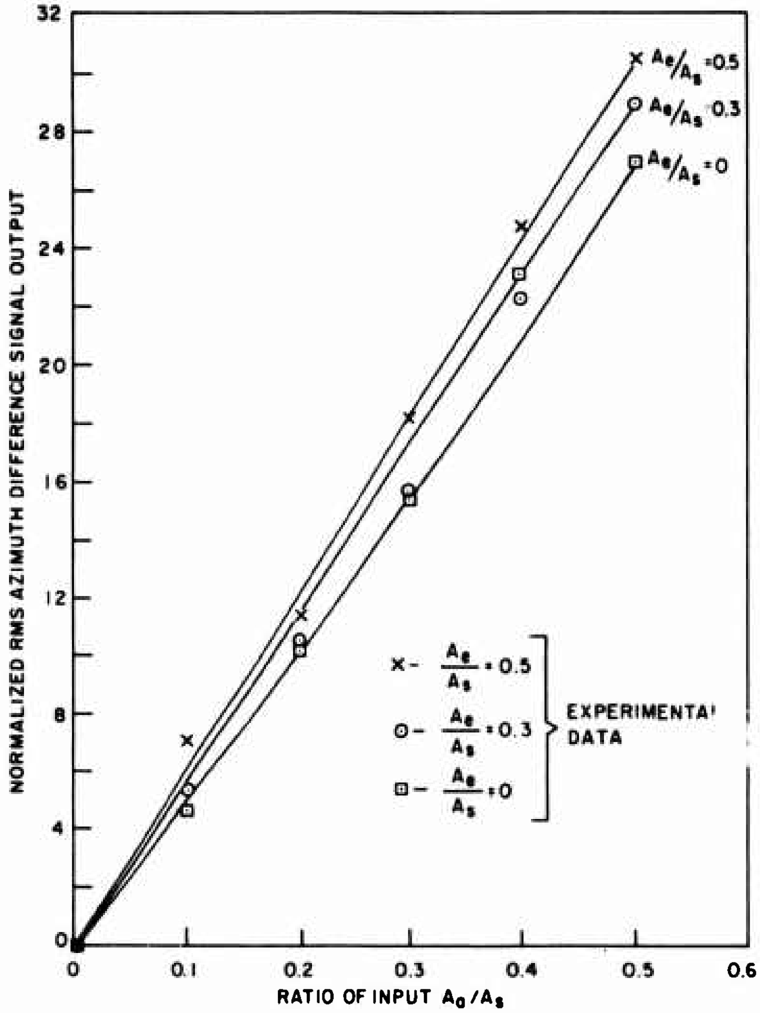

Fig. 7 – Illustration of cross modulation effect in azimuth channel for unequal spacing of signal channels (2:1)

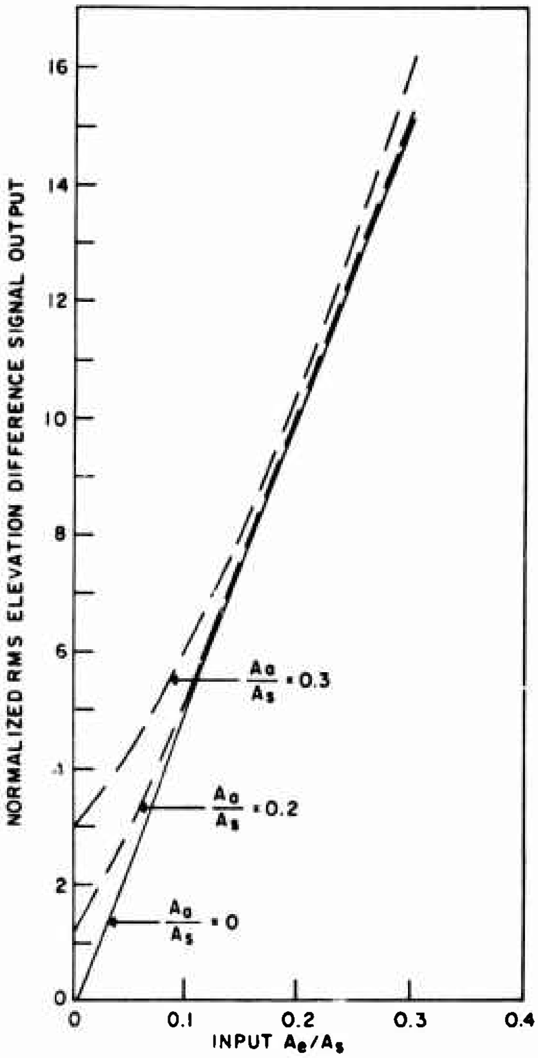

Fig. 8 — Illustration of cross-modulation effect in elevation channel for unequal spacing of signal channels (2:1)

The formula from Eq. (A12) for the output at ω_e is:

$$L_3(t)|\omega_e = \frac{4T}{\pi u^{1/2}} \left\{ \left[\frac{A_e}{A_s} - \frac{A_e}{2uA_s}\left(\frac{A_a^2}{A_s^2}+1\right) + \frac{3A_e}{4u^2A_s}\left(\frac{A_e^2}{A_s^2}+\frac{3A_a^2}{A_s^2}\right) \right.\right.$$

$$\left. - \frac{15A_e}{16u^3A_s}\left(\frac{A_e^2}{A_s^2}+\frac{2A_a^2}{A_s^2}\right)\right] \cos\,(\omega_e t + \phi_e)$$

$$+ \left[\frac{3A_a^2}{8u^2A_s^2} - \frac{15A_a^2}{16u^3A_s^2}\left(\frac{A_a^2}{3A_s^2}+\frac{3A_e^2}{A_s^2}\right) + \frac{35A_a^2}{32u^4A_s^2}\left(\frac{A_a^2}{A_s^2}+\frac{3A_e^2}{A_s^2}\right)\right] \cos\,(\omega_e t + \phi_0 + \phi_e)$$

$$\left. + \left[-\frac{15A_a^2A_e^2}{8u^3A_s^4} + \frac{105A_a^2A_e^2}{64u^4A_s^4}\right] \cos\,(\omega_e t + \phi_e - \phi_0)\right\}. \tag{19}$$

The results are presented in Fig. 8, which shows a residual amount of cross modulation in excess of that in the previous example.

EXPERIMENTAL SIMULATION

In order to verify the theoretical results, measurement of the cross-modulation effects in a simulated Scamp signal processor was made in the laboratory. This was done on a cw basis in the audio-frequency range, using commercial generators as signal sources. A counter was used to monitor the signal frequencies. For the equally spaced case, three input frequencies of 2500, 3000, and 3500 cps were selected, and the output centered on 2500 cps was read on a wave analyzer. The experimental results are shown on Fig. 3 for input ratios of the interfering channel of $A_e/A_s = 0$, 0.2, and 0.4. It is noted that the agreement with theory is excellent. A check was made to determine whether the final phase detector, which was not included in either the theory or simulation, would have an appreciable effect on the cross modulation. Both theory and experiment indicate that this final phase detector has no significant effect on the output error.

Tests were also made using the 2:1 channel spacing with input frequencies of 2000, 3000, and 3500 cps. The experimental points shown in Fig. 7 provide satisfactory agreement with the theory for this case.

SUMMARY

The theory of the Scamp signal processor is developed for a complete monopulse system with three channels: a sum signal or reference channel, and azimuth and elevation difference signal channels. Formulas are developed giving the difference-channel outputs for three selected arrangements and spacings of these channels. The worst channel arrangement, from the standpoint of cross modulation, is that of the three equally spaced contiguous channels, with the sum channel in the center. In this situation the cross modulation is intolerably great on both difference channels.

A second and far better arrangement for contiguous equally spaced channels is to place the sum channel at one end, with the two difference channels adjacent. This eliminates any interference in that difference channel nearest the reference but leaves a small residual cross modulation in the more remote difference channel. It is not felt that this would be intolerable for some applications.

A third arrangement using a 2:1 channel spacing yields similar results, but the residual cross modulation in the remote channel is greater than in case two. Wider and unequal separation of the three channels could result in less or no cross modulation if amplifier bandwidth permitted this mechanization. However, it would seem that such a move would tend to reduce one of the main advantages of the Scamp concept.

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Appendix

EXPANSION OF TERMS IN LIMITER OUTPUT

INTRODUCTION

In order to simplify the expansion of Eq. (2) the following notation will be adopted:

$$a = \frac{A_a}{A_s},\ e = \frac{A_e}{A_s},\ r_1 = 2a,\ r_2 = 2ae,\ r_3 = 2e$$

$$\alpha_1 = (\omega_s - \omega_a)\ t + (\phi_s - \phi_a) \quad c_1 = \cos \alpha_1$$

$$\alpha_2 = (\omega_a - \omega_e)\ t + (\phi_a - \phi_e) \quad c_2 = \cos \alpha_2$$

$$\alpha_3 = (\omega_s - \omega_e)\ t + (\phi_s - \phi_e) \quad c_3 = \cos \alpha_3$$

$$F = a \cos (\omega_a t + \phi_a) + \cos (\omega_s t + \phi_s) + e \cos (\omega_e t + \phi_e)$$

$$u = a^2 + 1 + e^2 \text{ and } v = r_1 c_1 + r_2 c_2 + r_3 c_3. \tag{A1}$$

Applying the definitions in Eq. (A1) to Eq. (2) we have

$$L(t) = \frac{4T}{\pi} F(u + v)^{-1/2} = \frac{4T}{\pi}\, u^{-1/2}\, F\left(1 + \frac{v}{u}\right)^{-1/2}. \tag{A2}$$

Whenever $v < u$ we can expand $(1 + v/u)^{-1/2}$ by the binomial theorem

$$L(t) = \frac{4T}{\pi}\, u^{-1/2}\, F\left(1 - \frac{\iota}{2u} + \frac{3v^2}{8u^2} - \frac{5v^3}{16u^3} + \frac{35v^4}{128u^4} - \frac{63v^5}{256u^5} + \dots\right). \tag{A3}$$

The powers of v expand in terms of the r's, c's, and α's as follows:

$$v = r_1 c_1 + r_2 c_2 + r_3 c_3 = r_1 \cos \alpha_1 + r_2 \cos \alpha_2 + r_3 \cos \alpha_3, \tag{A4}$$

$$\begin{aligned} v^2 &= r_1^2 c_1^2 + r_2^2 c_2^2 + r_3^2 c_3^2 + 2\, r_1 r_2 c_1 c_2 + 2\, r_1 r_3 c_1 c_3 + 2\, r_2 r_3 c_2 c_3 \\ &= \frac{1}{2}(r_1^2 + r_2^2 + r_3^2) + \frac{1}{2}(r_1^2 \cos 2\alpha_1 + r_2^2 \cos 2\alpha_2 + r_3^2 \cos 2\alpha_3) \\ &\quad + r_1 r_2 \cos (\alpha_1 - \alpha_2) + r_1 r_2 \cos \alpha_3 + r_1 r_3 \cos \alpha_2 \\ &\quad + r_1 r_3 \cos (\alpha_1 + \alpha_3) + r_2 r_3 \cos (\alpha_2 + \alpha_3) + r_2 r_3 \cos \alpha_1, \end{aligned} \tag{A5}$$

$$\begin{aligned} v^3 &= r_1^3 c_1^3 + r_2^3 c_2^3 + r_3^3 c_3^3 + 6\, r_1 r_2 r_3\, c_1 c_2 c_3 + 3\, r_1^2 c_1^2 r_2 c_2 \\ &\quad + 3 r_1 c_1 r_2^2 c_2^2 + 3 r_1^2 c_1^2 r_3 c_3 + 3 r_1 c_1 r_3^2 c_3^2 + 3 r_2^2 c_2^2 r_3 c_3 \\ &\quad + 3 r_2 c_2 r_3^2 c_3^2 \end{aligned}$$

$$= \frac{3}{4}(r_1^3 \cos \alpha_1 + r_2^3 \cos \alpha_2 + r_3^3 \cos \alpha_3) + \frac{1}{4}(r_1^3 \cos 3\alpha_1$$

$$+ r_2^3 \cos 3\alpha_2 + r_3^3 \cos 3\alpha_3) + \frac{3}{2} r_1 r_2 r_3 (1 + \cos 2\alpha_1$$

$$+ \cos 2\alpha_2 + \cos 2\alpha_3) + \frac{3}{2}(r_1^2 r_2 \cos \alpha_2 + r_1 r_2^2 \cos \alpha_1$$

$$+ r_1^2 r_3 \cos \alpha_3 + r_1 r_3^2 \cos \alpha_1 + r_2^2 r_3 \cos \alpha_3 + r_2 r_3^2 \cos \alpha_2)$$

$$+ \frac{3}{4}[r_1^2 r_2 \cos(2\alpha_1 - \alpha_2) + r_1^2 r_2 \cos(\alpha_1 + \alpha_3)$$

$$+ r_1 r_2^2 \cos(2\alpha_2 - \alpha_1) + r_1 r_2^2 \cos(\alpha_2 + \alpha_3) + r_1^2 r_3 \cos(\alpha_1 - \alpha_2)$$

$$+ r_1^2 r_3 \cos(2\alpha_1 + \alpha_3) + r_1 r_3^2 \cos(\alpha_2 + \alpha_3) + r_1 r_3^2 \cos(2\alpha_3 + \alpha_1)$$

$$+ r_2^2 r_3 \cos(\alpha_1 - \alpha_2) + r_2^2 r_3 \cos(2\alpha_2 + \alpha_3) + r_2 r_3^2 \cos(\alpha_1 + \alpha_3)$$

$$+ r_2 r_3^2 \cos(2\alpha_3 + \alpha_2)], \tag{A6}$$

$$v^4 = r_1^4 c_1^4 + r_2^4 c_2^4 + r_3^4 c_3^4 + 4r_1^3 r_2 c_1^3 c_2 + 4r_1^3 r_3 c_1^3 c_3$$

$$+ 4r_1 r_2^3 c_1 c_2^3 + 4r_1 r_3^3 c_1 c_3^3 + 4r_2 r_3^3 c_2 c_3^3 + 4r_2^3 r_3 c_2^3 c_3$$

$$+ 6r_1^2 r_2^2 c_1^2 c_2^2 + 6r_1^2 r_3^2 c_1^2 c_3^2 + 6r_2^2 r_3^2 c_2^2 c_3^2$$

$$+ 12 r_1^2 r_2 r_3 c_1^2 c_2 c_3 + 12 r_1 r_2^2 r_3 c_1 c_2^2 c_3 + 12 r_1 r_2 r_3^2 c_1 c_2 c_3^2$$

$$= \frac{3}{8}(r_1^4 + r_2^4 + r_3^4) + \frac{1}{2}(r_1^4 \cos 2\alpha_1 + r_2^4 \cos 2\alpha_2 + r_3^4 \cos 2\alpha_3)$$

$$+ \frac{1}{8}(r_1^4 \cos 4\alpha_1 + r_2^4 \cos 4\alpha_2 + r_3^4 \cos 4\alpha_3) + \frac{3}{2}[r_1^3 r_2 \cos(\alpha_1 - \alpha_2)$$

$$+ r_1^3 r_2 \cos \alpha_3 + r_1^3 r_3 \cos \alpha_2 + r_1^3 r_3 \cos(\alpha_3 + \alpha_1)$$

$$+ r_1 r_2^3 \cos(\alpha_1 - \alpha_2) + r_1 r_2^3 \cos \alpha_3 + r_1 r_3^3 \cos \alpha_2$$

$$+ r_1 r_3^3 \cos(\alpha_3 + \alpha_1) + r_2 r_3^3 \cos \alpha_1 + r_2 r_3^3 \cos(\alpha_3 + \alpha_2)$$

$$+ r_2^3 r_3 \cos \alpha_1 + r_2^3 r_3 \cos(\alpha_3 + \alpha_2)] + \frac{1}{2}[r_1^3 r_2 \cos(3\alpha_1 - \alpha_2)$$

$$+ r_1^3 r_2 \cos(2\alpha_1 + \alpha_3) + r_1^3 r_3 \cos(2\alpha_1 - \alpha_2) + r_1^3 r_3 \cos(3\alpha_1 + \alpha_3)$$

$$+ r_1 r_2^3 \cos(3\alpha_2 - \alpha_1) + r_1 r_2^3 \cos(2\alpha_2 + \alpha_3) + r_1 r_3^3 \cos(3\alpha_3 - \alpha_1)$$

$$+ r_1 r_3^3 \cos(3\alpha_3 + \alpha_1) + r_2 r_3^3 \cos(3\alpha_3 - \alpha_2) + r_2 r_3^3 \cos(3\alpha_3 + \alpha_2)$$

$$+ r_2^3 r_3 \cos(2\alpha_2 - \alpha_1) + r_2^3 r_3 \cos(3\alpha_2 + \alpha_3)] + \frac{3}{2}(r_1^2 r_2^2$$

$$+ r_1^2 r_3^2 + r_2^2 r_3^2) + \frac{3}{2}(r_1^2 r_2^2 \cos 2\alpha_1 + r_1^2 r_2^2 \cos 2\alpha_2$$

$$+ r_1^2 r_3^2 \cos 2\alpha_1 + r_1^2 r_3^2 \cos 2\alpha_3 + r_2^2 r_3^2 \cos 2\alpha_2 + r_2^2 r_3^2 \cos 2\alpha_3)$$

$$+ \frac{3}{4}[r_1^2 r_2^2 \cos 2(\alpha_2 - \alpha_1) + r_1^2 r_2^2 \cos 2\alpha_3 + r_1^2 r_3^2 \cos 2\alpha_2$$

$$+ r_1^2 r_3^2 \cos 2(\alpha_3 + \alpha_1) + r_2^2 r_3^2 \cos 2\alpha_1 + r_2^2 r_3^2 \cos 2(\alpha_3 + \alpha_2)]$$

$$+ 3\,[r_1^2 r_2 r_3 \cos \alpha_1 + r_1^2 r_2 r_3 \cos(\alpha_3 + \alpha_2) + r_1 r_2^2 r_3 \cos \alpha_2$$

$$+ r_1 r_2^2 r_3 \cos(\alpha_3 + \alpha_1) + r_1 r_2 r_3^2 \cos(\alpha_2 - \alpha_1) + r_1 r_2 r_3^2 \cos \alpha_3]$$

$$+ \frac{3}{2}[r_1^2 r_2 r_3 \cos \alpha_1 + r_1^2 r_2 r_3 \cos 3\alpha_1 + r_1^2 r_2 r_3 \cos(2\alpha_2 - \alpha_1)$$

$$+ r_1^2 r_2 r_3 \cos(2\alpha_3 + \alpha_1) + r_1 r_2^2 r_3 \cos \alpha_2 + r_1 r_2^2 r_3 \cos 3\alpha_2$$

$$+ r_1 r_2^2 r_3 \cos(2\alpha_1 - \alpha_2) + r_1 r_2^2 r_3 \cos(2\alpha_3 + \alpha_2) + r_1 r_2 r_3^2 \cos(2\alpha_1 + \alpha_3)$$

$$+ r_1 r_2 r_3^2 \cos(2\alpha_2 + \alpha_3) + r_1 r_2 r_3^2 \cos \alpha_3 + r_1 r_2 r_3^2 \cos 3\alpha_3]. \tag{A7}$$

Rather than evaluating all of the terms in Eqs. (A4) through (A7) in terms of the ω's, let us select only those terms which (when multiplied by F) possess either a frequency ω_a or ω_e, for each arrangement of channels considered. In Table A1 are listed the terms arising from Fv, Fv^2, *etc.*

1. EQUISPACED SYMMETRICAL CHANNELS (FIG. 2)

Here $\omega_s = \omega_s$

$\omega_a = \omega_s - \delta$

$\omega_e = \omega_s + \delta.$

In Table A1 the terms containing frequency combinations which yield ω_a are underlined, and those which yield ω_e are marked by an asterisk. Collecting terms on ω_a, the output from Eq. (A3) at that angular frequency is (up to fourth degree in the amplitude coefficients, a and e)

$$L_1(t)|\omega_a = \frac{4T}{\pi u^{1/2}}\left\{\left[a - \frac{a}{2u}(1 + e^2) + \frac{3a}{4u^2}(a^2 + 3e^2)\right.\right.$$

$$\left. - \frac{15a}{16u^3}(a^2 + 2e^2)\right] \cos(\omega_a t + \phi_a) + \left[-\frac{e}{2u} + \frac{3e}{8u^2}(4a^2 + e^2)\right.$$

$$\left. - \frac{15e}{16u^3}(2a^2 + e^2)\right] \cos(\omega_a t + \phi_1) + \left[\frac{9a^2 e}{8u^2} - \frac{15a^2 e}{16u^3}\right] \cos(\omega_a t + \phi_2)$$

$$\left. - \frac{15ae^2}{16u^3} \cos(\omega_a t + \phi_3)\right\}, \tag{A8}$$

where $\phi_1 \equiv 2\phi_s - \phi_e$, $\phi_2 \equiv -(2\phi_s - 2\phi_a - \phi_e)$ and $\phi_3 \equiv 4\phi_s - \phi_a - 2\phi_e$. Note the phase change in ϕ_2 to keep ω_a positive.

TABLE A1
Terms Arising from Fv, Fv^2, Fv^3, and Fv^4

Product	Amplitudes and Frequencies Resulting from Product
$2F\cos\alpha_1$	$a[\omega_s]$, $a[\omega_s-2\omega_a]$, $l[2\omega_s-\omega_a]$,* $\underline{l[\omega_a]}$, $^{\dagger\triangle}$ $e[\omega_s-\omega_a+\omega_e]$, $e[\omega_s-\omega_a-\omega_e]$
$2F\cos\alpha_2$	$a[\omega_e]$,$^{*\circ\square}$ $a[\omega_e-2\omega_a]$, $l[\omega_s+\omega_a-\omega_e]$,$^{\circ}$ $l[\omega_s-\omega_a+\omega_e]$, $\underline{e[\omega_a]}$, $^{\dagger\triangle}$ $e[\omega_a-2\omega_e]$
$2F\cos\alpha_3$	$a[\omega_s+\omega_a-\omega_e]$,$^{\circ}$ $a[\omega_s-\omega_a-\omega_e]$, $\underline{l[2\omega_s-\omega_e]}$, $l[\omega_e]$,$^{*\circ\square}$ $e[\omega_s]$, $e[\omega_s-2\omega_e]^{\dagger}$
$2F\cos 2\alpha_1$	$a[2\omega_s-\omega_a]$,* $a[2\omega_s-3\omega_a]$, $l[\omega_s-2\omega_a]$, $l[3\omega_s-2\omega_a]$,$^{\square}$ $\underline{e[2\omega_s-2\omega_a-\omega_e]}$, $e[2\omega_s-2\omega_a+\omega_e]$
$2F\cos 2\alpha_2$	$a[\omega_a-2\omega_e]$, $a[3\omega_a-2\omega_e]$, $l[\omega_s+2\omega_a-2\omega_e]$,† $l[\omega_s-2\omega_a+2\omega_e]$, $e[2\omega_a-3\omega_e]$, $e[2\omega_a-\omega_e]$
$2F\cos 2\alpha_3$	$a[2\omega_s-2\omega_e-\omega_a]$,* $a[2\omega_s-2\omega_e+\omega_a]$, $l[\omega_s-2\omega_e]$,† $l[3\omega_s-2\omega_e]$, $\underline{e[2\omega_s-\omega_e]}$, $e[2\omega_s-3\omega_e]$
$2F\cos(\alpha_1-\alpha_2)$	$a[\omega_s-\omega_a+\omega_e]$, $a[\omega_s-3\omega_a+\omega_e]$, $l[2\omega_s-2\omega_a+\omega_e]$, $l[2\omega_a-\omega_e]$, $e[\omega_s-2\omega_a]$, $e[\omega_s-2\omega_a+2\omega_e]$
$2F\cos(\alpha_1+\alpha_3)$	$\underline{a[2\omega_s-\omega_e]}$, $\underline{a[2\omega_s-2\omega_a-\omega_e]}$, $l[\omega_s-\omega_a-\omega_e]$, $l[3\omega_s-\omega_a-\omega_e]$,$^{\triangle}$ $e[2\omega_s-\omega_a]$,* $e[2\omega_s-\omega_a-2\omega_e]^{*}$
$2F\cos(\alpha_2+\alpha_3)$	$a[\omega_s-2\omega_e]$,† $a[2\omega_a+\omega_s-2\omega_e]$,† $l[\omega_a+2\omega_s-2\omega_e]$, $l[\omega_a-2\omega_e]$, $e[\omega_a+\omega_s-\omega_e]$,$^{\circ}$ $e[\omega_a+\omega_s-3\omega_e]^{\circ}$
$2F\cos 3\alpha_1$	$a[3\omega_s-4\omega_a]$, $a[3\omega_s-2\omega_a]$,$^{\square}$ $l[4\omega_s-3\omega_a]$, $l[2\omega_s-3\omega_a]$, $e[3\omega_s-3\omega_a-\omega_e]$,$^{\triangle}$ $e[3\omega_s-3\omega_a+\omega_e]$
$2F\cos 3\alpha_2$	$a[2\omega_a-3\omega_e]$, $a[4\omega_a-3\omega_e]$, $l[\omega_s+3\omega_a-3\omega_e]$, $l[3\omega_a-\omega_s-3\omega_e]$, $e[3\omega_a-4\omega_e]$, $e[3\omega_a-2\omega_e]$
$2F\cos 3\alpha_3$	$a[3\omega_s-3\omega_e-\omega_a]$, $a[3\omega_s-3\omega_e+\omega_a]$, $l[2\omega_s-3\omega_e]$, $l[4\omega_s-3\omega_e]$ $e[3\omega_s-2\omega_e]$, $e[3\omega_s-4\omega_e]$
$2F\cos(2\alpha_1-\alpha_2)$	$a[2\omega_s-2\omega_a+\omega_e]$, $a[2\omega_s-4\omega_a+\omega_e]$, $l[3\omega_s-3\omega_a+\omega_e]$, $l[\omega_s-3\omega_a+\omega_e]$, $e[2\omega_s-3\omega_a+2\omega_e]$, $e[2\omega_s-3\omega_a]$
$2F\cos(2\alpha_2-\alpha_1)$	$a[2\omega_a-\omega_s-2\omega_e]$, $a[4\omega_a-\omega_s-2\omega_e]$, $l[3\omega_a-2\omega_s-2\omega_e]$, $l[3\omega_a-2\omega_e]$, $e[3\omega_a-\omega_s-\omega_e]$, $e[3\omega_a-\omega_s-3\omega_e]$
$2F\cos(2\alpha_1+\alpha_3)$	$a[3\omega_s-\omega_a-\omega_e]$,$^{\triangle}$ $a[3\omega_s-3\omega_a-\omega_e]$,$^{\triangle}$ $l[4\omega_s-2\omega_a-\omega_e]$,* $\underline{l[2\omega_s-2\omega_a-\omega_e]}$, $e[3\omega_s-2\omega_a-2\omega_e]$,$^{\square}$ $e[3\omega_s-2\omega_a]^{\square}$
$2F\cos(2\alpha_3+\alpha_1)$	$a[3\omega_s-2\omega_a-2\omega_e]$,$^{\square}$ $a[3\omega_s-2\omega_e]$, $\underline{l[4\omega_s-\omega_a-2\omega_e]}$, $l[2\omega_s-\omega_a-2\omega_e]$,* $e[3\omega_s-\omega_a-\omega_e]$,$^{\triangle}$ $e[3\omega_s-\omega_a-3\omega_e]$

NOTE: The symbols $*$, $\dagger$, $\triangle$, $\circ$, $\square$, and the underlining are explained in this appendix.

(Table Continues)

TABLE A1 (Continued)
Terms Arising from Fv, Fv^2, Fv^3, and Fv^4

Product	Amplitudes and Frequencies Resulting from Product
$2F\cos(2\alpha_2+\alpha_3)$	$a[3\omega_a+\omega_s-3\omega_e]$, $a[\omega_a+\omega_s-3\omega_e],^{\circ}$ $1[2\omega_a+2\omega_s-3\omega_e],^{\circ}$ $1[2\omega_a-3\omega_e]$, $e[2\omega_a+\omega_s-4\omega_e]$, $e[2\omega_a+\omega_s-2\omega_e]^{\dagger}$
$2F\cos(2\alpha_3+\alpha_2)$	$a[2\omega_s+2\omega_a-3\omega_e],^{\circ}$ $a[2\omega_s-3\omega_e],^{\circ}$ $1[3\omega_s+\omega_a-3\omega_e]$, $1[\omega_s+\omega_a-3\omega_e],^{\circ}$ $e[2\omega_s+\omega_a-4\omega_e],^{\dagger}$ $e[2\omega_s+\omega_a-2\omega_e]$
$2F\cos 4\alpha_1$	$a[4\omega_s-3\omega_a]$, $a[4\omega_s-5\omega_a]$, $1[3\omega_s-4\omega_a]$, $1[5\omega_s-4\omega_a]$, $e[4\omega_s-4\omega_a-\omega_e]$, $e[4\omega_s-4\omega_a+\omega_e]$
$2F\cos 4\alpha_2$	$a[5\omega_a-4\omega_e]$, $a[3\omega_a-4\omega_e]$, $1[4\omega_a-4\omega_e-\omega_s]$, $1[4\omega_a-4\omega_e+\omega_s]$, $e[4\omega_a-5\omega_e]$, $e[4\omega_a-3\omega_e]$
$2F\cos 4\alpha_3$	$a[4\omega_s-4\omega_e+\omega_a]$, $a[4\omega_s-4\omega_e-\omega_a]$, $1[5\omega_s-4\omega_e]$, $1[3\omega_s-4\omega_e]$, $e[4\omega_s-3\omega_e]$, $e[4\omega_s-5\omega_e]$
$2F\cos(3\alpha_1-\alpha_2)$	$a[3\omega_s-5\omega_a+\omega_e]$, $a[3\omega_s-3\omega_a+\omega_e]$, $1[4\omega_s-4\omega_a+\omega_e]$, $1[2\omega_s-4\omega_a+\omega_e]$, $e[3\omega_s-4\omega_a+2\omega_e]$, $e[3\omega_s-4\omega_a]$
$2F\cos(3\alpha_1+\alpha_3)$	$a[4\omega_s-4\omega_a-\omega_e]$, $a[4\omega_s-2\omega_a-\omega_e],^{*}$ $1[5\omega_s-3\omega_a-\omega_e]$, $1[3\omega_s-3\omega_a-\omega_e],^{\triangle}$ $\underline{e[4\omega_s-3\omega_a-2\omega_e]}$, $e[4\omega_s-3\omega_a]$
$2F\cos(3\alpha_2-\alpha_1)$	$a[5\omega_a-\omega_s-3\omega_e]$, $a[3\omega_a-\omega_s-3\omega_e]$, $1[4\omega_a-2\omega_s-3\omega_e]$, $1[4\omega_a-3\omega_e]$, $e[4\omega_a-\omega_s-4\omega_e]$, $e[4\omega_a-\omega_s-2\omega_e]$
$2F\cos(3\alpha_3-\alpha_1)$	$a[2\omega_s-3\omega_e],^{\circ}$ $a[2\omega_s-3\omega_e+2\omega_a],^{\circ}$ $1[3\omega_s-3\omega_e+\omega_a]$, $1[\omega_s-3\omega_e+\omega_a],^{\circ}$ $e[2\omega_s-4\omega_e+\omega_a],^{\dagger}$ $e[2\omega_s-2\omega_e+\omega_a]$
$2F\cos(3\alpha_3+\alpha_1)$	$a[4\omega_s-3\omega_e]$, $a[4\omega_s-3\omega_e-2\omega_a],^{*}$ $1[3\omega_s-3\omega_e-\omega_a]$, $1[5\omega_s-3\omega_e-\omega_a]$, $e[4\omega_s-4\omega_e-\omega_a]$, $\underline{e[4\omega_s-2\omega_e-\omega_a]}$
$2F\cos(3\alpha_3-\alpha_2)$	$a[3\omega_s-2\omega_e]$, $a[3\omega_s-2\omega_e-2\omega_a]^{\square}_{,}$ $\underline{1[4\omega_s-2\omega_e-\omega_a]}$, $1[2\omega_s-2\omega_e-\omega_a],^{*}$ $e[3\omega_s-3\omega_e-\omega_a]$, $e[3\omega_s-\omega_e-\omega_a]^{\triangle}$
$2F\cos(3\alpha_3+\alpha_2)$	$a[3\omega_s-4\omega_e]$, $a[3\omega_s-4\omega_e+2\omega_a]$, $1[4\omega_s-4\omega_e+\omega_a]$, $1[2\omega_s-4\omega_e+\omega_a],^{\dagger}$ $e[3\omega_s-5\omega_e+\omega_a]$, $e[3\omega_s-3\omega_e+\omega_a]$
$2F\cos(3\alpha_2+\alpha_3)$	$a[4\omega_a+\omega_s-4\omega_e]$, $a[2\omega_a+\omega_s-4\omega_e]$, $1[3\omega_a-4\omega_e]$, $1[3\omega_a+2\omega_s-4\omega_e],^{\dagger}$ $e[3\omega_a+\omega_s-5\omega_e]$, $e[3\omega_a+\omega_s-3\omega_e]$
$2F\cos 2(\alpha_2-\alpha_1)$	$a[5\omega_a-2\omega_s-2\omega_e]$, $a[3\omega_a-2\omega_s-2\omega_e]$, $1[4\omega_a-3\omega_s-2\omega_e]$, $1[4\omega_a-\omega_s-2\omega_e]$, $e[4\omega_a-2\omega_s-3\omega_e]$, $e[4\omega_a-2\omega_s-\omega_e]$
$2F\cos 2(\alpha_3+\alpha_1)$	$\underline{a[4\omega_s-3\omega_a-2\omega_e]}$, $\underline{a[4\omega_s-\omega_a-2\omega_e]}$, $1[3\omega_s-2\omega_a-2\omega_e]^{\square}_{,}$ $1[5\omega_s-2\omega_a-2\omega_e]$, $e[4\omega_s-2\omega_a-3\omega_e],^{*}$ $e[4\omega_s-2\omega_a-\omega_e]^{*}$
$2F\cos 2(\alpha_3+\alpha_2)$	$a[3\omega_a+2\omega_s-4\omega_e],^{\dagger}$ $a[\omega_a+2\omega_s-4\omega_e],^{\dagger}$ $1[2\omega_a+3\omega_s-4\omega_e]$, $1[2\omega_a+\omega_s-4\omega_e]$, $e[2\omega_a+2\omega_s-5\omega_e],^{\circ}$ $e[2\omega_a+2\omega_s-3\omega_e]^{\circ}$

Collecting terms on ω_e yields the same expression as Eq. (A8), except $a \to e$, $e \to a$, $\omega_a \to \omega_e$, $\phi_a \to \phi_e$, and $\phi_e \to \phi_a$ so that $\phi_1 \equiv 2\,\phi_s - \phi_a$, *etc.* One would expect this symmetry, since the ω_a and ω_e channels are symmetrically spaced about the center sum channel, ω_s. If we let $\phi_0 = 2\,\phi_s - \phi_a - \phi_e$ for the ω_a channel, then $\phi_1 = \phi_a + \phi_0$, $\phi_2 = +\,\phi_a - \phi_0$ and $\phi_3 = \phi_a' + 2\,\phi_0$.

2. EQUISPACED NONSYMMETRICAL CHANNELS (FIG. 4)

Here $\omega_a = \omega_a$

$$\omega_e = \omega_a + \delta$$

$$\omega_s = \omega_a + 2\,\delta$$

In Table A1 the terms containing frequency combinations which yield ω_a are marked by a † and those which yield ω_e by a O. If we let $\phi_0 = \phi_s + \phi_a - 2\,\phi_e$, then the output from Eq. (A3) at ω_a is:

$$L_2(t)|\omega_a = \frac{4T}{\pi u^{1/2}}\Bigg\{\left[a - \frac{a(1+e^2)}{2u} + \frac{3a(a^2+3e^2)}{4u^2} - \frac{15a(a^2+2e^2)}{16u^3}\right]\cos\,(\omega_a t + \phi_a)$$

$$+\left[-\frac{e^2}{2u} + \frac{3e^2}{8u^2}\,(4a^2+1) - \frac{15e^2}{16u^3}\,(5a^2+e^2) + \frac{35e^2}{32u^4}\,(3a^2+e^2)\right]\cos\,(\omega_a t + \phi_a - \phi_0)$$

$$+\left[\frac{9a^2e^2}{8u^2} - \frac{45a^2e^2}{16u^3} + \frac{105a^2e^2}{64u^4}\cos\,(\omega_a t + \phi_a + \phi_0)\right]\Bigg\}. \tag{A9}$$

Again, if we let $\phi_0 = \phi_s + \phi_a - 2\phi_e$, then the output from Eq. (A3) at ω_e is:

$$L_2(t)|\omega_e = \frac{4T}{\pi u^{1/2}}\Bigg\{\left[e - \frac{e(a^2+1)}{2u} + \frac{3e(e^2+3a^2)}{4u^2} - \frac{15e(e^2+2a^2)}{16u^3}\right]\cos\,(\omega_e t + \phi_e)$$

$$+\left[-\frac{ae}{u} + \frac{3ae(a^2+e^2+1)}{4u^2} - \frac{15ae(e^2+a^2)}{4u^3} + \frac{105ae(a^2+e^2)}{32u^4}\right]\cos\,(\omega_e t + \phi_e + \phi_0)$$

$$+\left[\frac{3ae^3}{4u^2} - \frac{15ae^3}{8u^3} + \frac{35ae^3}{32u^4}\right]\cos\,(\omega_e t + \phi_e - \phi_0)\Bigg\}. \tag{A10}$$

Once more note the change in sign of phases to keep ω_a and ω_e positive.

3. UNEQUALLY SPACED CHANNELS (FIG. 6)

Here $\omega_s = \omega_s$

$$\omega_a = \omega_s - \delta$$

$$\omega_e = \omega_s + 2\delta$$

In Table A1 the terms containing frequency combinations which yield ω_a are marked by a △ and those which yield ω_e by a □. If we let $\phi_0 = 3\phi_s - 2\phi_a - \phi_e$, then the output from (A3) at ω_a is:

$$L_3(t)|\omega_a = \frac{4T}{\pi u^{1/2}}\Bigg\{\left[a - \frac{a(1+e^2)}{2u} + \frac{3a(a^2+3e^2)}{4u^2} - \frac{15a(a^2+2e^2)}{16u^3}\right]\cos\,(\omega_a t + \phi_a)$$

$$+\left[\frac{3ae}{4u^2} - \frac{15ae(a^2+e^2)}{8u^3} + \frac{105ae(a^2+e^2)}{32u^4}\right]\cos\,(\omega_a t + \phi_a + \phi_0)$$

$$+\left[-\frac{5a^3e}{4u^3} + \frac{35a^3e}{32u^4}\right]\cos\,(\omega_a t + \phi_a - \phi_0)\Bigg\}. \tag{A11}$$

Again if we let $\phi_0 = 3\phi_s - 2\phi_a - \phi_e$, then the output from Eq. (A3) at ω_e is:

$$L_3(t)|\omega_e = \frac{4T}{\pi u^{1/2}} \left\{ \left[e - \frac{e(a^2+1)}{2u} + \frac{3e(e^2+3a^2)}{4u^2} - \frac{15e(e^2+2a^2)}{16u^3} \right] \cos\,(\omega_e t + \phi_e) \right.$$

$$+ \left[\frac{3a^2}{8u^2} - \frac{15a^2}{16u^3} \left(\frac{a^2}{3} + 3e^2 \right) + \frac{35a^2}{32u^4}\,(a^2+3e^2) \right] \cos\,(\omega_e t + \phi_0 + \phi_e)$$

$$\left. + \left[-\frac{15a^2e^2}{8u^3} + \frac{105a^2e^2}{64u^4} \right] \cos\,(\omega_e t + \phi_e - \phi_0) \right\}. \qquad \text{(A12)}$$

Note the phase sign change to keep ω_a and ω_e positive.

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13. ABSTRACT

When two or more signals of different frequencies are fed simultaneously into the input of a limiter, cross-modulation products exist in the output. Such a situation occurs in the Scamp (Single-Channel Monopulse Processor) signal processor when the sum signal and both difference signals (azimuth and elevation) of a monopulse system are processed in a common limiter. The resultant cross modulation can produce deleterious effects, even though the spectra of the three input signals do not overlap. By the use of approximation methods, formulas are developed for the limiter outputs, showing the dependence of the cross-modulation terms on the input amplitudes and phases of both difference channels. Curves are presented illustrating the amount of cross modulation for several arrangements of channel spacing. The largest errors occur when the difference channels are symmetrically located on either side of the reference channel. The errors are substantially reduced by an unsymmetrical or a noncontiguous channel orientation. Experimental results on a simulated Scamp processor agree favorably with the theory.

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